ANALYSIS OF A TWO-REGIME STOCHASTIC CAR-FOLLOWING MODEL: EXPLAINING CAPACITY DROP AND OSCILLATION INSTABILITIES

TU XU, PHD STUDENT
JORGE A. LAVAL, PHD

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TRB

Outline

- Two-regime car-following models
- The congestion term
- The free-flow term
- Analysis/Advantages of the model
- Conclusion

Two regime car-following models

Parameter estimation of deterministic car-following models:

$$x_j(t) = F(\mathbf{x}; \Theta) + \epsilon \qquad \text{(Hoogendoorn and Ossen, 2005)}$$

$x_j(t)$ is the vehicle position time t (or speed or acceleration),
F is a deterministic car-following model with parameters $\Theta = (\theta_1, \theta_2, \ldots)$,
$\mathbf{x}$=$\{x_j(t_i)\}$ are the trajectory data points for vehicle j at times t_i,
ϵ is a normal random variable with mean of zero.

The additive error may not be appropriate for car-following models.

Two regime car-following models

$$X_j(t) = \min\{ \underbrace{Y(\mathbf{x};\Theta)}_{congestion}, \underbrace{Z(\mathbf{x};\Theta)}_{free-flow} \}$$

If random processes Y and Z are normally distributed:

$$Y \sim N(\mu_Y, \sigma_Y), \qquad Z \sim N(\mu_Z, \sigma_Z)$$

One can show that(Nadarajah and Kotz, 2008):

$$f(x;\Theta) = \frac{1}{2\sqrt{2\pi}} \left(\frac{1}{\sigma_Z} e^{-\frac{(\mu_Z - x)^2}{2\sigma_Z^2}} \operatorname{erfc}\left(\frac{x - \mu_Y}{\sqrt{2}\sigma_Y} \right) + \frac{1}{\sigma_Y} e^{-\frac{(\mu_Y - x)^2}{2\sigma_Y^2}} \operatorname{erfc}\left(\frac{x - \mu_Z}{\sqrt{2}\sigma_Z} \right) \right)$$

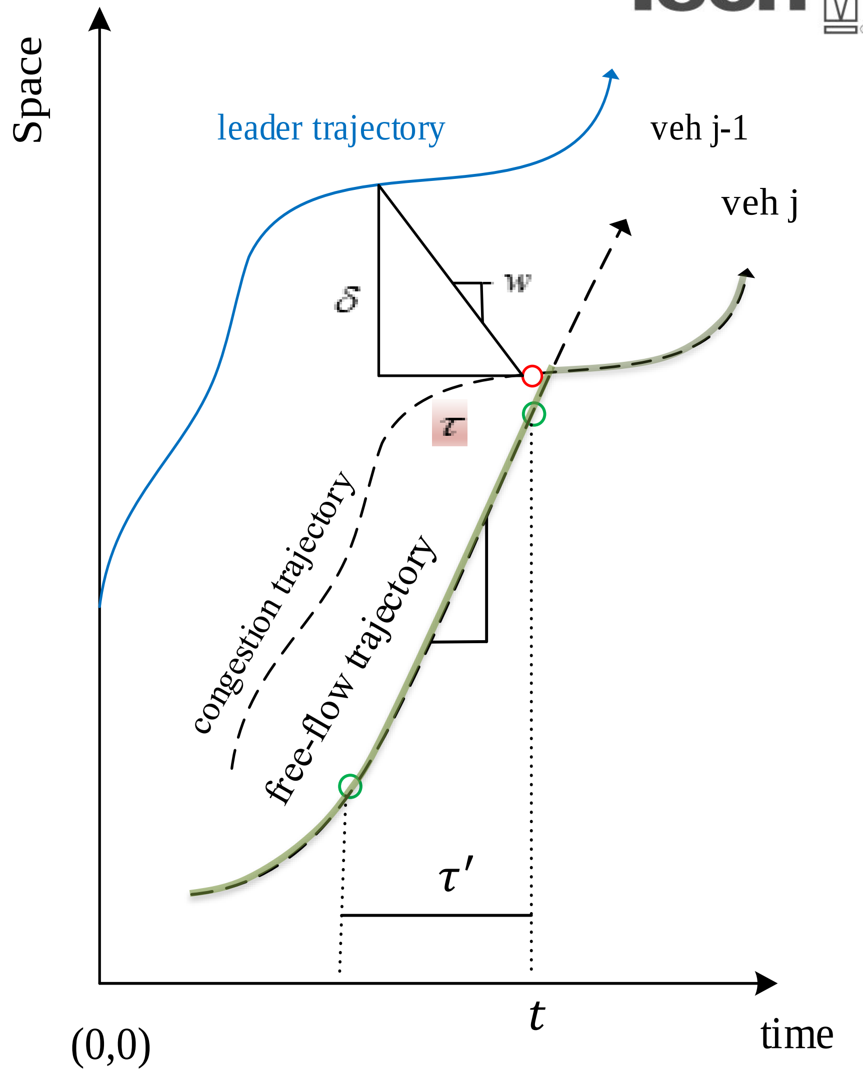
The model is analytical and lends itself nicely to be estimated using MLE.

Two regime car-following models

Example: Newell's car-following framework (Newell, 2002)

$$x_j(t) = \min\{\underbrace{x_{j-1}(t-\tau) - \delta}_{\text{congestion term } Y},\ \underbrace{x_j(t-\tau') + \xi_j(\tau')}_{\text{free-flow term } Z}\}$$

$x_j(t)$	the position of j th vehicle at time t
τ	wave trip time between two consecutive vehicle trajectories
δ	jam spacing (reciprocal of jam density)
$\xi_j(\tau')$	the desired displacement of j th vehicle in time τ'



The congestion term

$$Y = x_{j-1}(t-\tau) - \delta$$

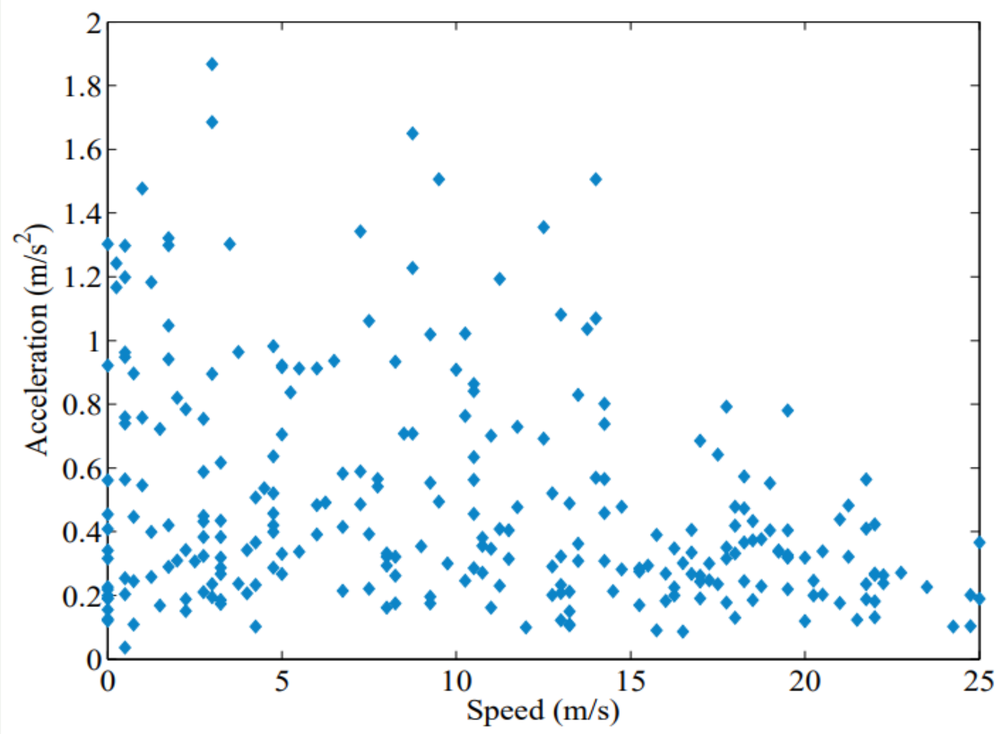
Parameters τ and δ can be assumed to follow the bivariate normal (BVN) distribution (Ahn et.al, 2003):

$$(\tau, \delta) \sim BVN(\mu_\tau, \mu_\delta, \sigma_\tau, \sigma_\delta, \rho),$$

such that the congestion term Y is normally distributed.

$$\begin{cases} \mu_Y = x_{j-1}(t-\mu_\tau) - \mu_\delta - a_{j-1}(t-\mu_\tau)\sigma_\tau^2/2, \\ \sigma_Y^2 = v_{j-1}^2(t-\mu_\tau)\sigma_\tau^2 + \sigma_\delta^2 + 2\rho v_{j-1}^2(t-\mu_\tau)\sigma_\tau\sigma_\delta, \end{cases}$$

The free-flow term



Note: Data taken for platoon leaders only when accelerating from a red light.

Figure 1 Relationship between the driver's desired **acceleration** and **vehicle speed**.

The free-flow term

(a) Proposed model

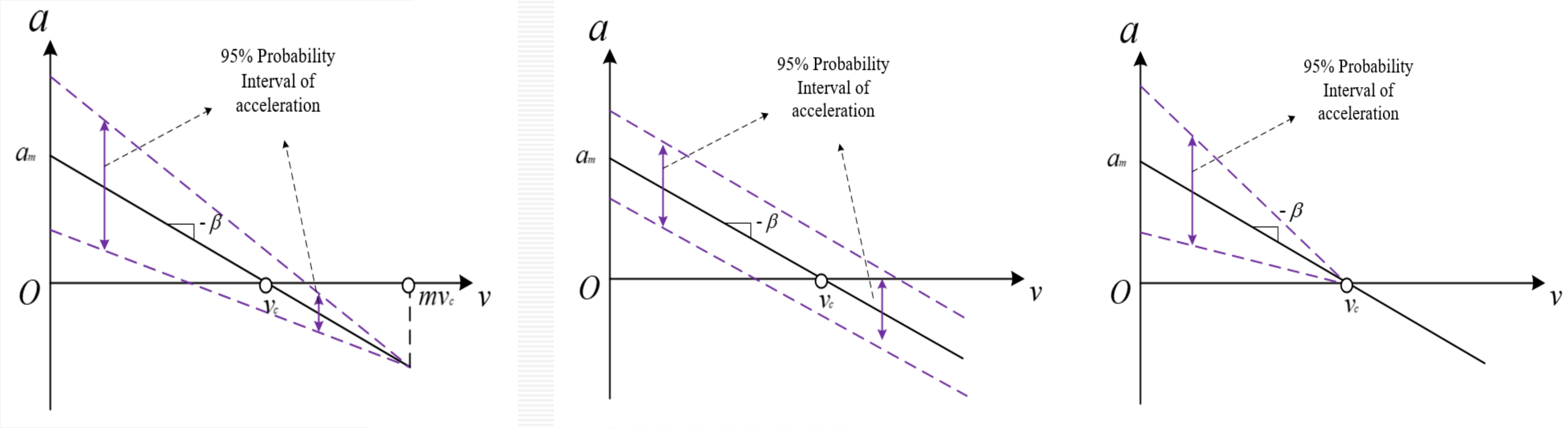
$m \geq 1$

(b) Brownian motion model

$m \gg 1$

(c) Geometric Brownian motion model

$m = 1$



Laval et.al, 2014

Yuan et.al, 2018

Figure 2 The relationship between 95% probability interval of **acceleration** and **vehicle speed** for different models

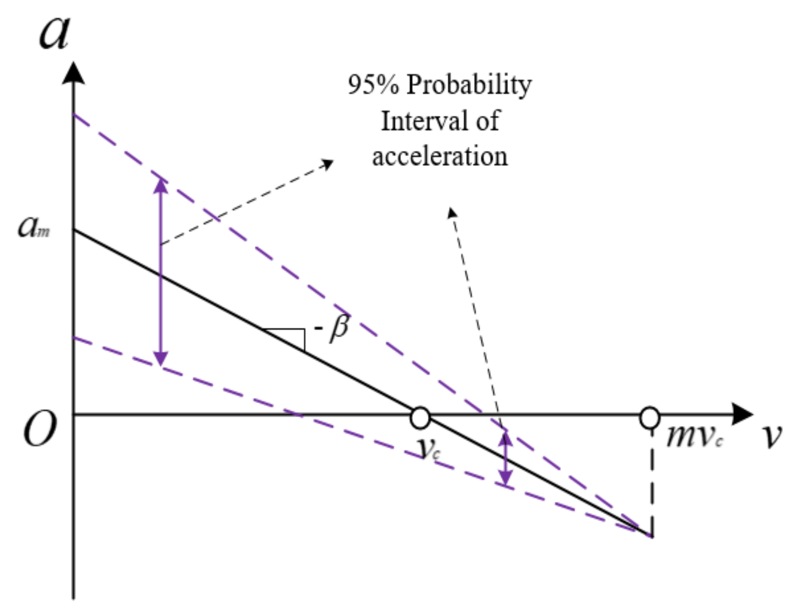
The free-flow term

$$\begin{cases} d\xi(t) = v(t)dt, \quad \xi(0) = 0, \\ dv(t) = (v_c - v(t))\beta dt + (mv_c - v(t))\sigma dW(t), \quad v(0) = v_0, \end{cases}$$

$W(t)$: a standard Brownian motion

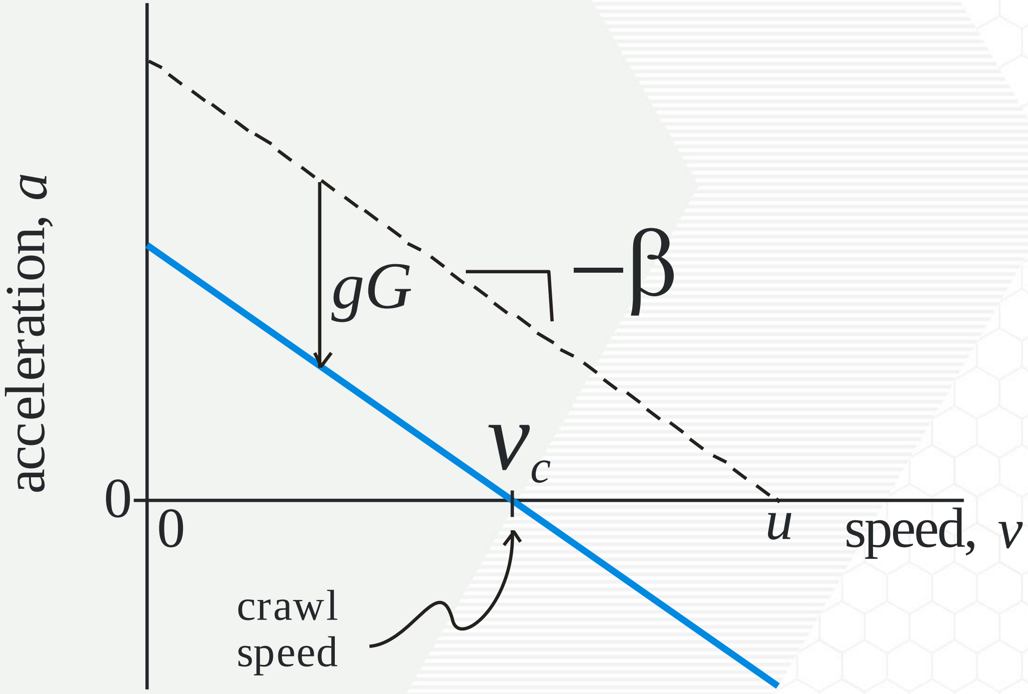
σ: diffusion coefficient

According to Central Limit Theorem, the distribution of $\xi(t)$ is normal such that the free-flow term Z follows normal distribution.



The free-flow term

On a 100G% upgrade



In the literature

$$a\big(v(t)\big) = \big(u - v(t)\big)\beta - gG$$

$$v_c = u - \frac{gG}{\beta}$$

Assumption:

$$a\big(v(t)\big) = \big(u - v(t)\big)\beta + \alpha gG$$
$$= \left(u + \alpha\frac{gG}{\beta} - v(t)\right)\beta$$

$$v_c = u + \alpha\frac{gG}{\beta}$$

The free-flow term

Define dimensionless variables with a tilde as follows:

$$\tilde{t} = \beta t, \qquad \tilde{v}(\tilde{t}) = v(\tilde{t})/v_c, \qquad \tilde{\xi}(\tilde{t}) = \beta\xi(\tilde{t})/v_c, \qquad \tilde{\sigma}^2 = \sigma^2/\beta.$$

The dimensionless form:

$$\begin{cases} \tilde{\xi}(\tilde{t}) = v(\tilde{t})dt, \qquad \tilde{\xi}(0) = 0, \\ \tilde{v}(\tilde{t}) = (1 - \tilde{v}(\tilde{t}))d\tilde{t} + (m - \tilde{v}(\tilde{t}))\tilde{\sigma}dW(\tilde{t}), \qquad \tilde{v}(0) = v_0/v_c, \end{cases}$$

Besides initial conditions, the only two non-observable parameters that drives this model are m and $\tilde{\sigma}$. The product of m and $\tilde{\sigma}$ has a big impact on the model performance.

Estimation of model parameters

$$
\begin{cases}
\mu_Y = x_{j-1}(t-\mu_\tau) - \mu_\delta - a_{j-1}(t-\mu_\tau)\sigma_\tau^2/2, \\
\sigma_Y^2 = v_{j-1}^2(t-\mu_\tau)\sigma_\tau^2 + \sigma_\delta^2 + 2\rho v_{j-1}^2(t-\mu_\tau)\sigma_\tau\sigma_\delta, \\
\mu_Z = x_j(t-\tau') + \mathrm{E}\left[\xi(\tau')\right], \\
\sigma_Z^2 = \mathrm{Var}\left[\xi(\tau')\right],
\end{cases}
$$

$$
Y \sim N(\mu_Y, \sigma_Y), \qquad Z \sim N(\mu_Z, \sigma_Z)
$$

$$
f(x;\Theta) = \frac{1}{2\sqrt{2\pi}} \left(\frac{1}{\sigma_Z} e^{-\frac{(\mu_Z - x)^2}{2\sigma_Z^2}} \operatorname{erfc}\left(\frac{x-\mu_Y}{\sqrt{2}\sigma_Y} \right) + \frac{1}{\sigma_Y} e^{-\frac{(\mu_Y - x)^2}{2\sigma_Y^2}} \operatorname{erfc}\left(\frac{x-\mu_Z}{\sqrt{2}\sigma_Z} \right) \right)
$$

such that we can use MLE to estimate the parameters:

$$
\Theta = (\mu_\tau, \mu_\delta, u, \beta, m, \tilde{\sigma}, \rho, \sigma_\tau, \sigma_\delta, \alpha)
$$

Data for estimation of model parameters

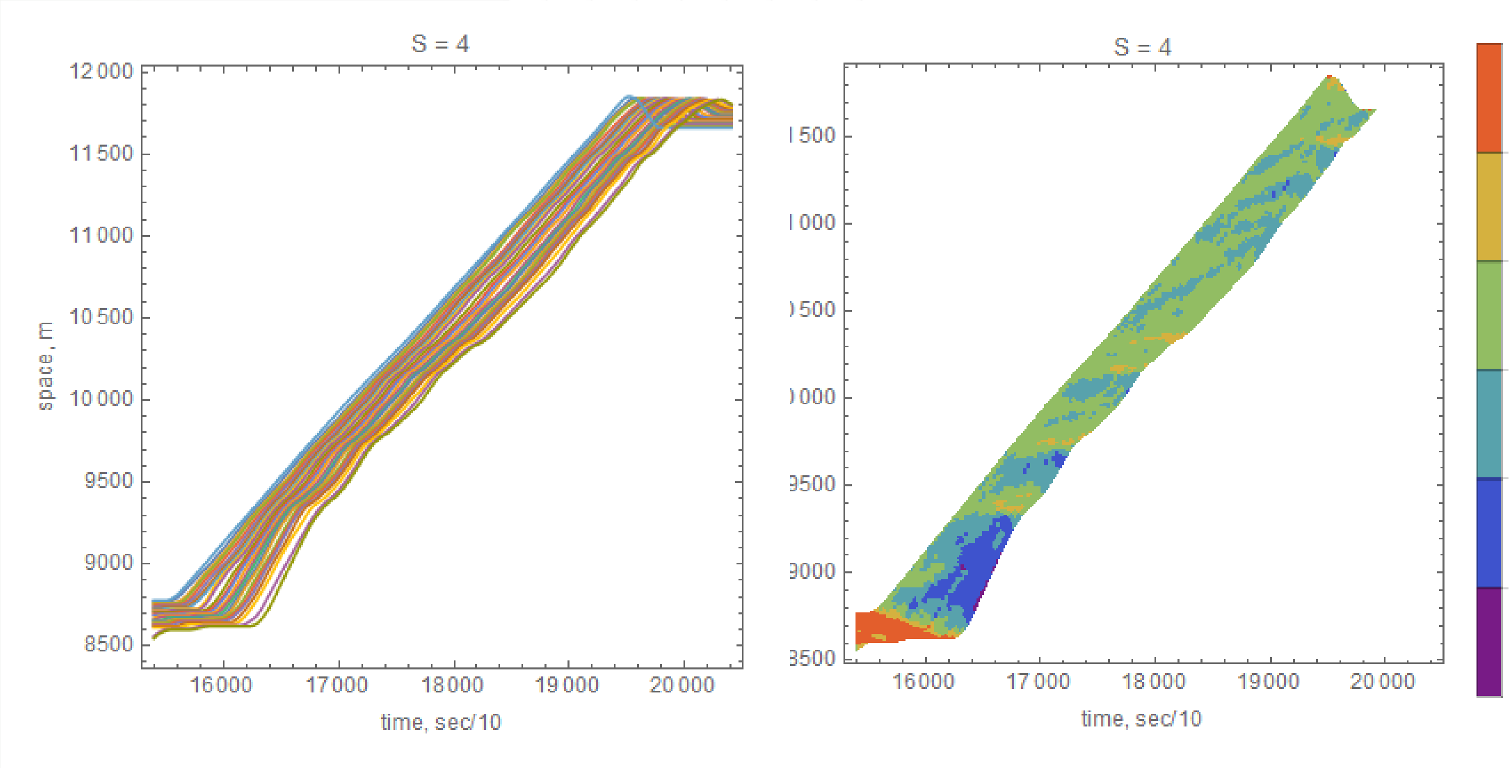


Figure 3 Example trajectory of car-following experiments used for parameter estimation (Jiang et.al 2014)

Estimation of model parameters

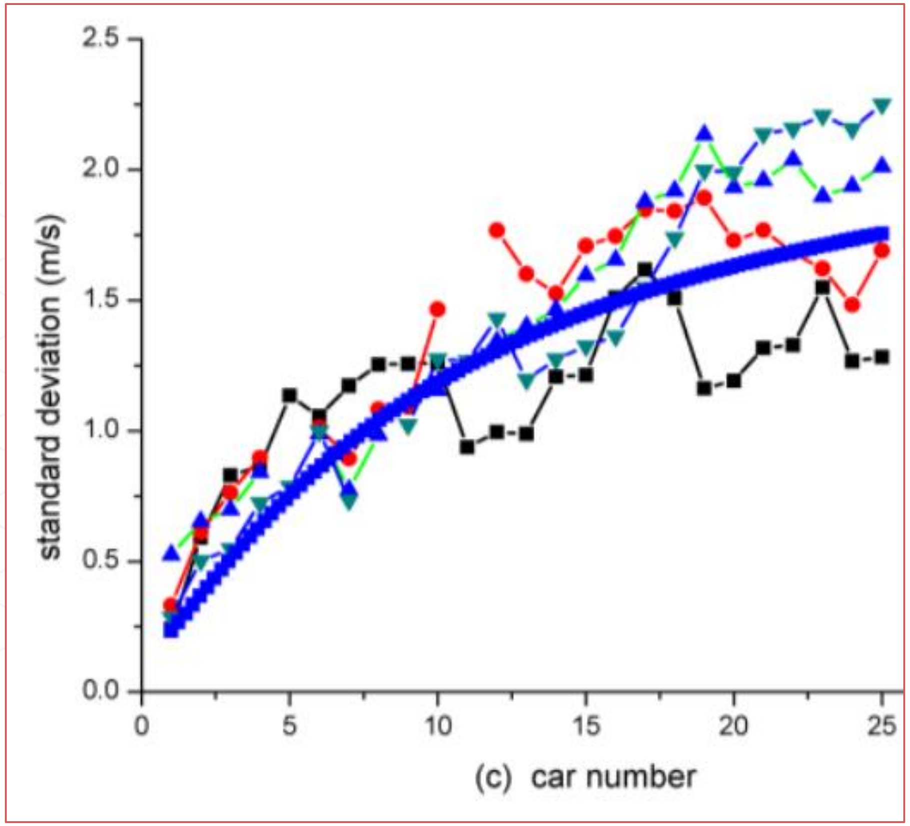
Table 1 Estimated parameter values

Parameter	mean value	t-stat
$\widehat{\mu_\tau}$	0.63	10.4
$\widehat{\mu_\delta}$	4.87	11.0
$\widehat{u}$	64.1	7.9
$\widehat{\beta}$	66.5	9.7
$\widehat{m}$	4.9	10.6
$\widehat{\tilde{\sigma}}$	0.052	11.9
$\widehat{\rho}$	-0.7	-15.5
$\widehat{\sigma_\tau}$	0.48	22.5
$\widehat{\sigma_\delta}$	2.17	37.7
$\widehat{\alpha}$	-0.59	-12.3

Analysis of the model

1. Acceleration process

2. Concave growth of platoon oscillation

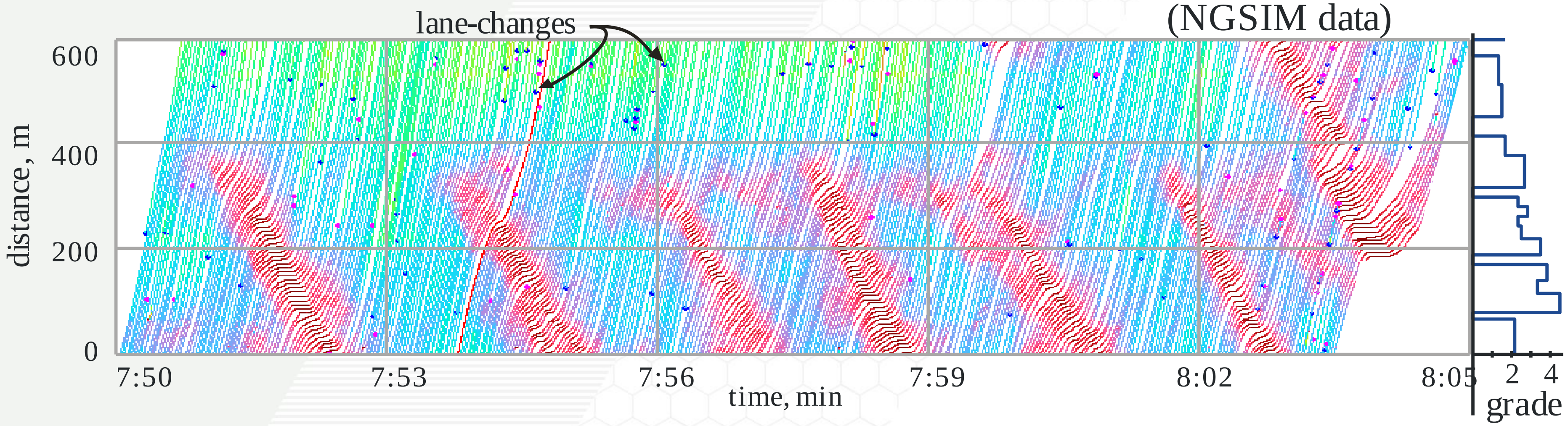


Analysis of the model

1. Acceleration process

2. Concave growth of platoon oscillation

3. Periodic oscillations at uphill segments



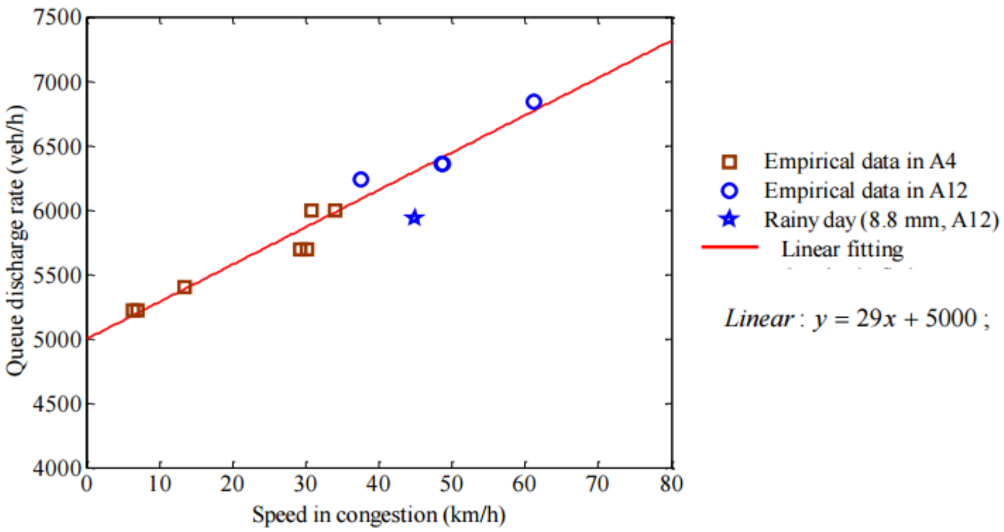
Analysis of the model

1. Acceleration process

2. Concave growth of platoon oscillation

3. Periodic oscillations at uphill segments

4. Speed-capacity relationship at bottlenecks



Yuan et.al, 2015

Analysis of the model

1. Acceleration process

2. Concave growth of platoon oscillation

3. Periodic oscillations at uphill segments

4. Speed-capacity relationship at bottlenecks

5. Prediction of vehicle speed distributions

1. The acceleration process

Georgia Tech

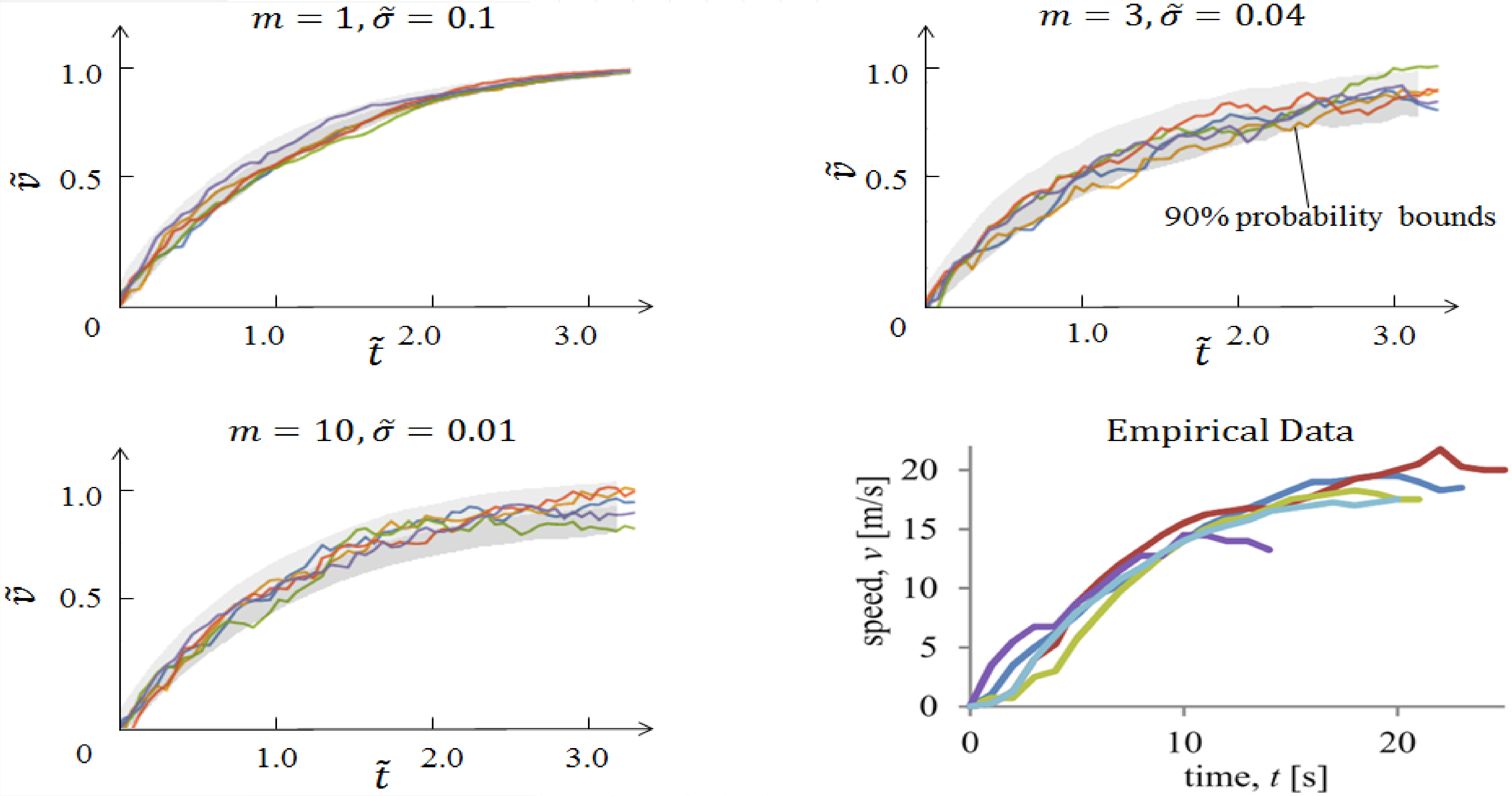
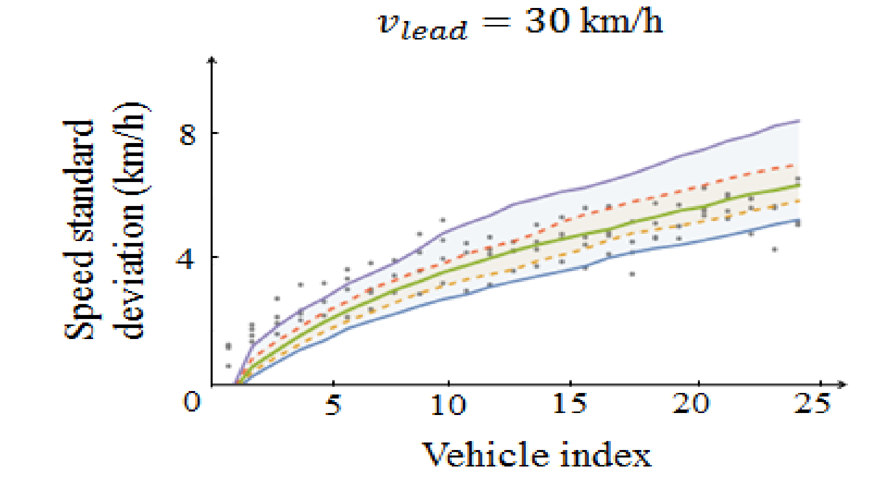
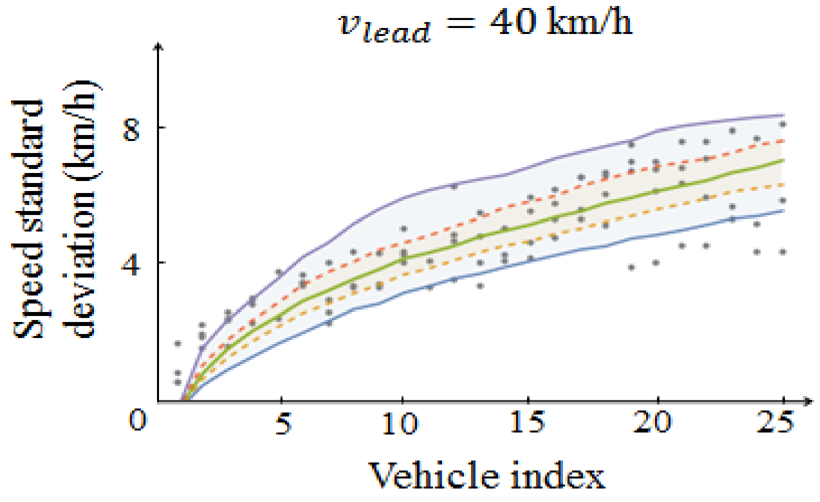
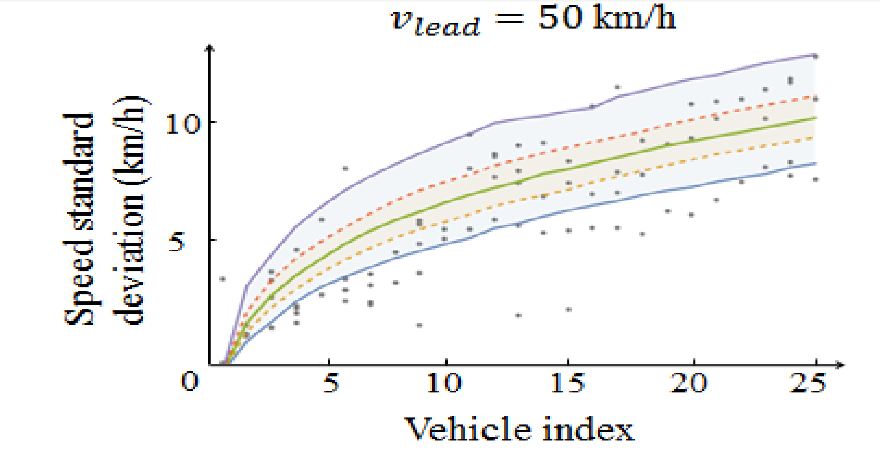


Figure 4 Five realizations along with the 90% probability bounds for the acceleration process starting from a complete stop along with empirical data

2. Concave growth of platoon oscillation

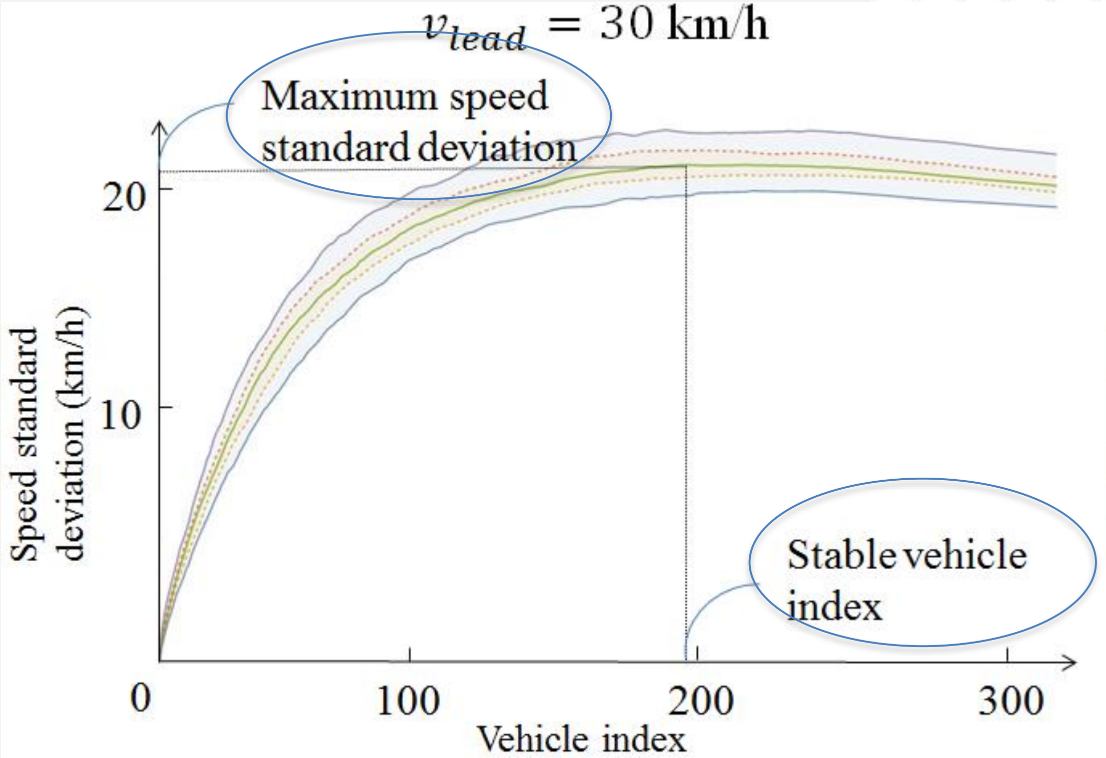


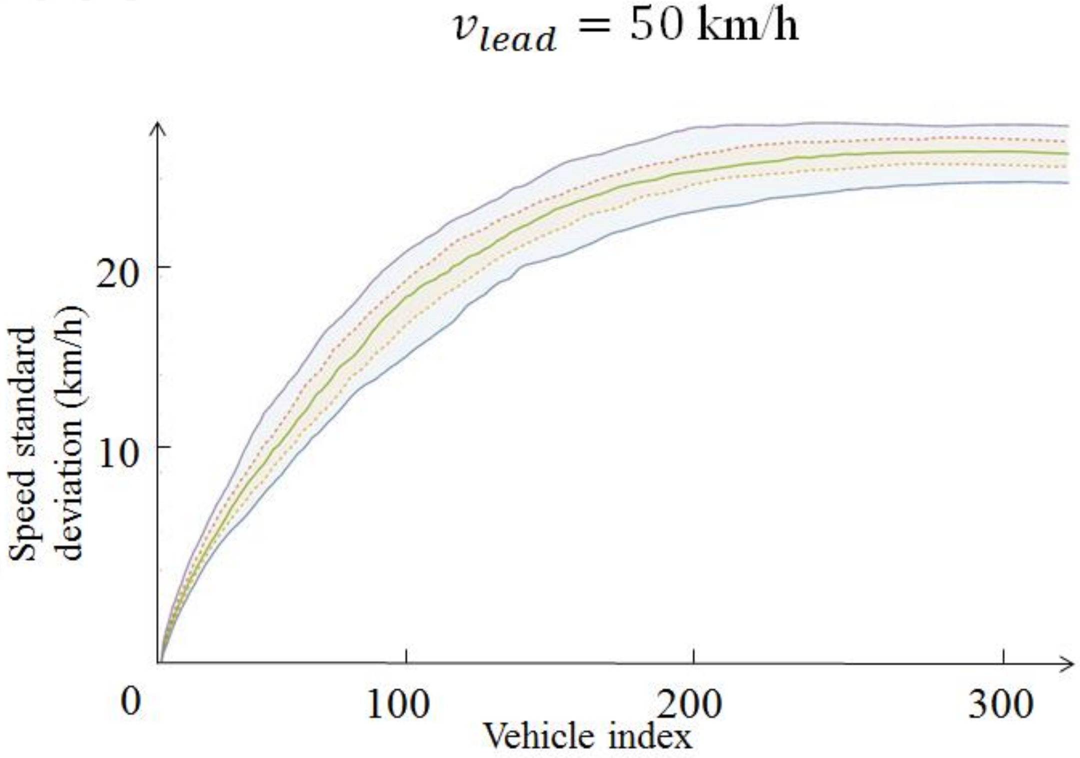




25-veh platoon simulation results compared to real data (Jiang et. Al, 2014)

2. Concave growth of platoon oscillation





300-veh platoon simulation

2. Concave growth of platoon oscillation

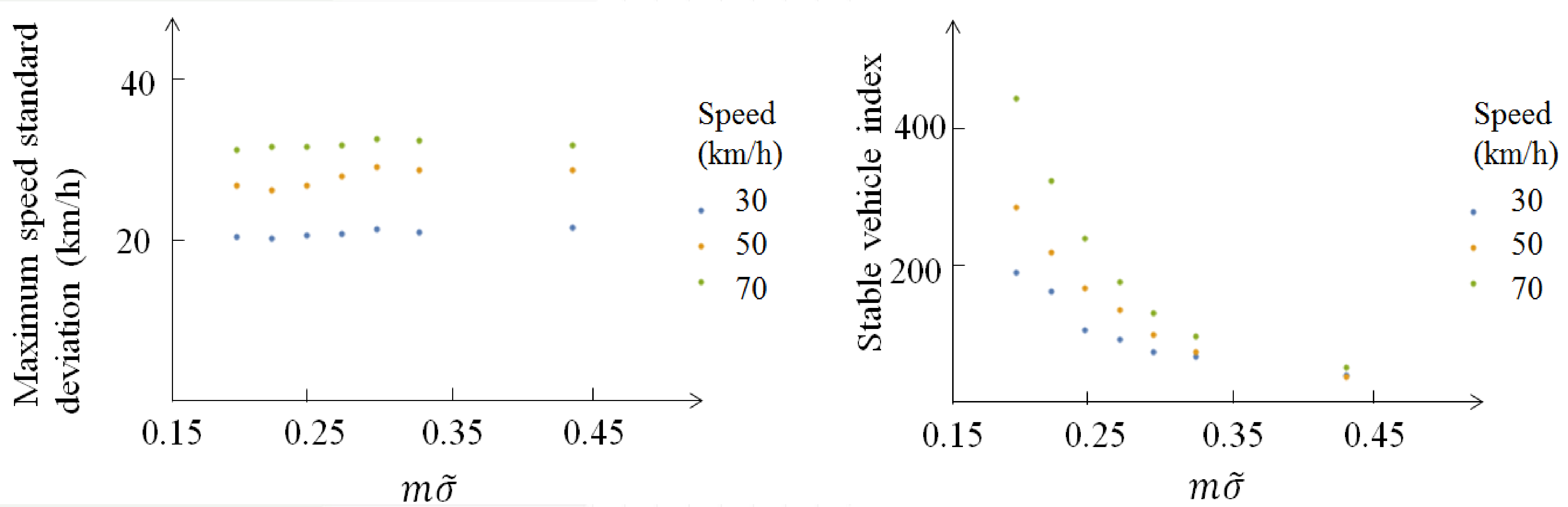


Figure 5 Relationship between the maximum speed variation, the stable vehicle index and the lead vehicle speed, model parameters.

3. Periodic oscillations at uphill segments

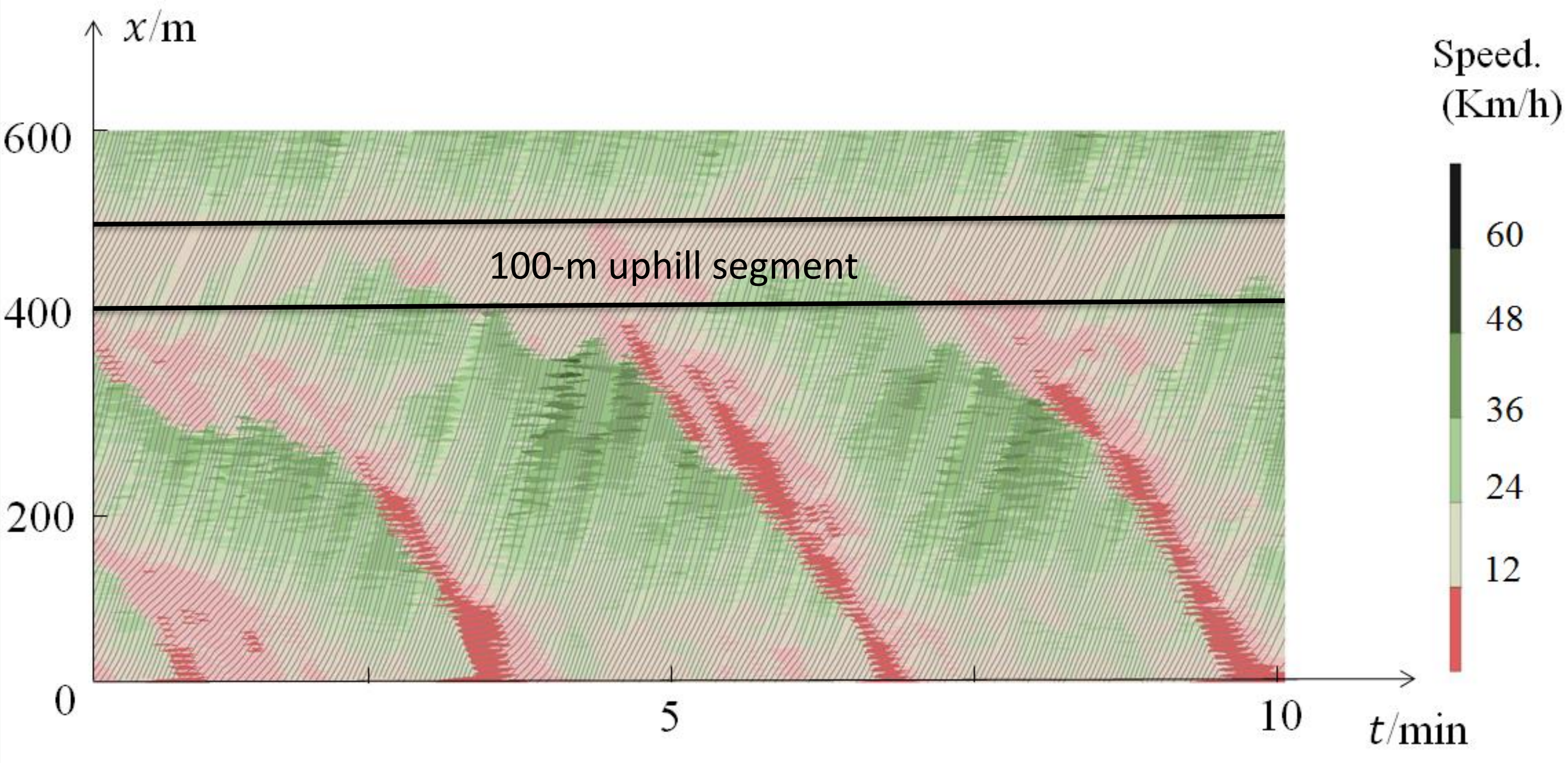
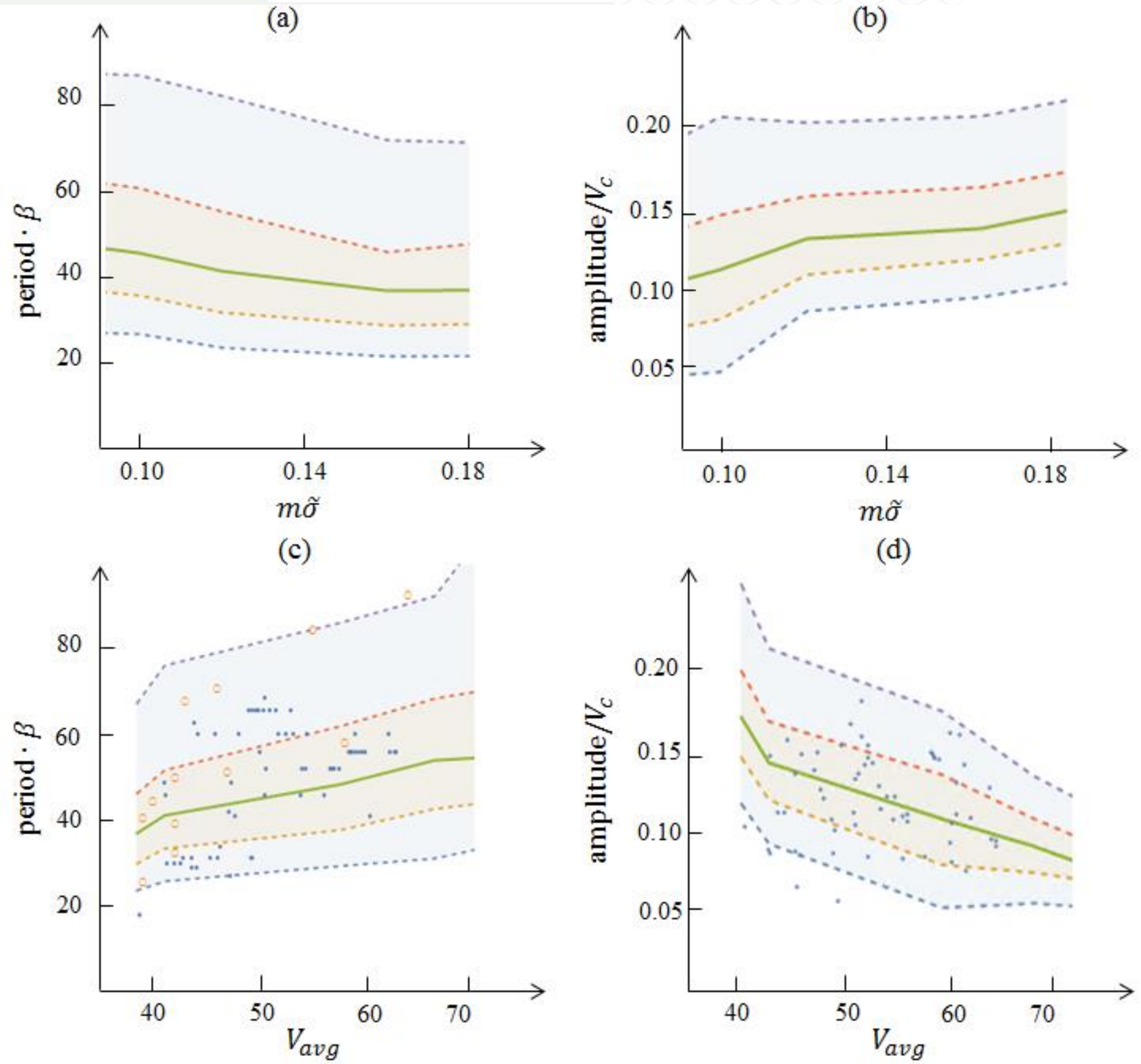


Figure 6 A typical simulation result with an upgrade of 5% and $m = 1.2$, $\tilde{\sigma} = 0.16$

3. Periodic oscillations at uphill segments



10000 simulation runs.

Empirical data:

- Treiber & Kesting 2012
- Knoop et.al 2012

4. Speed-capacity relationship at BNs

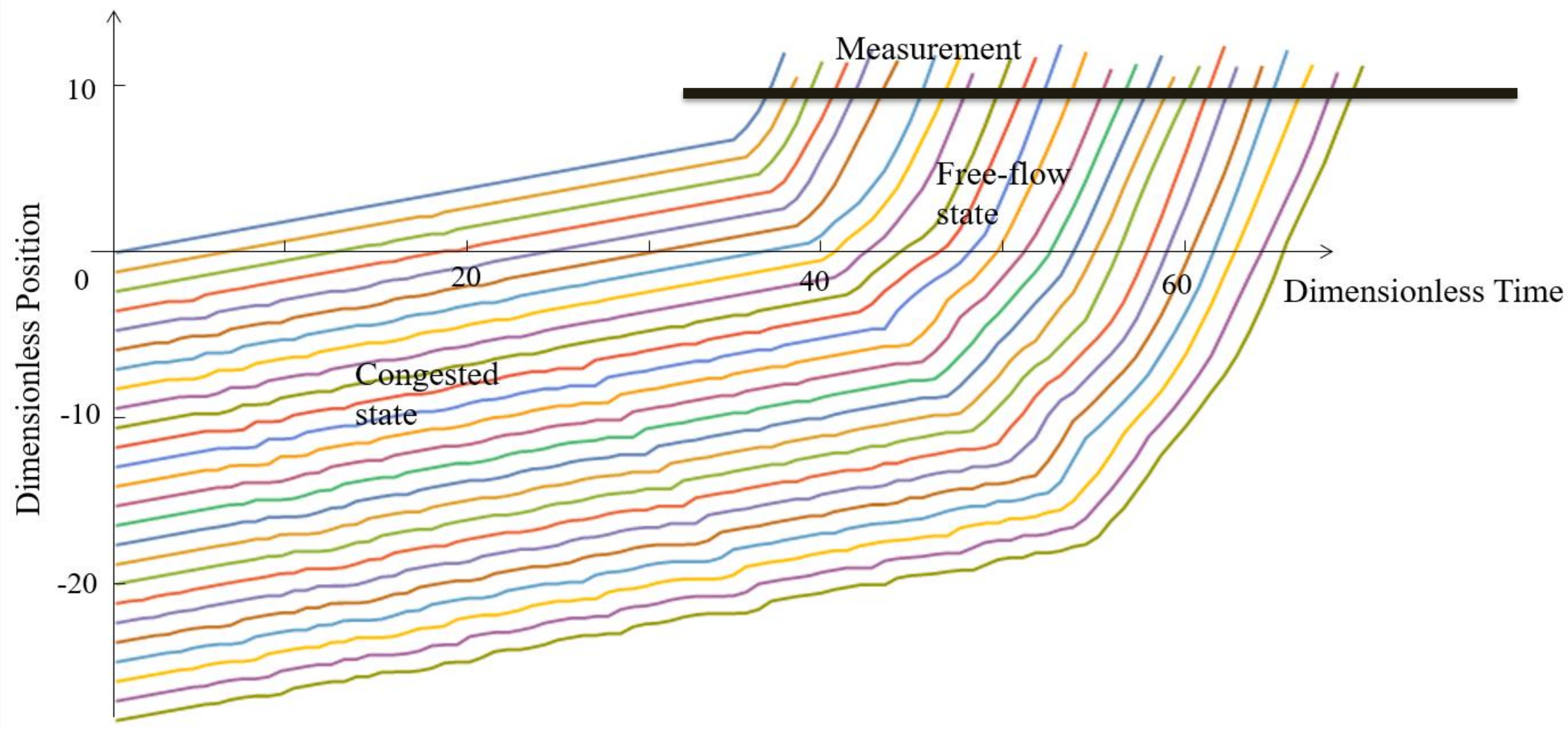
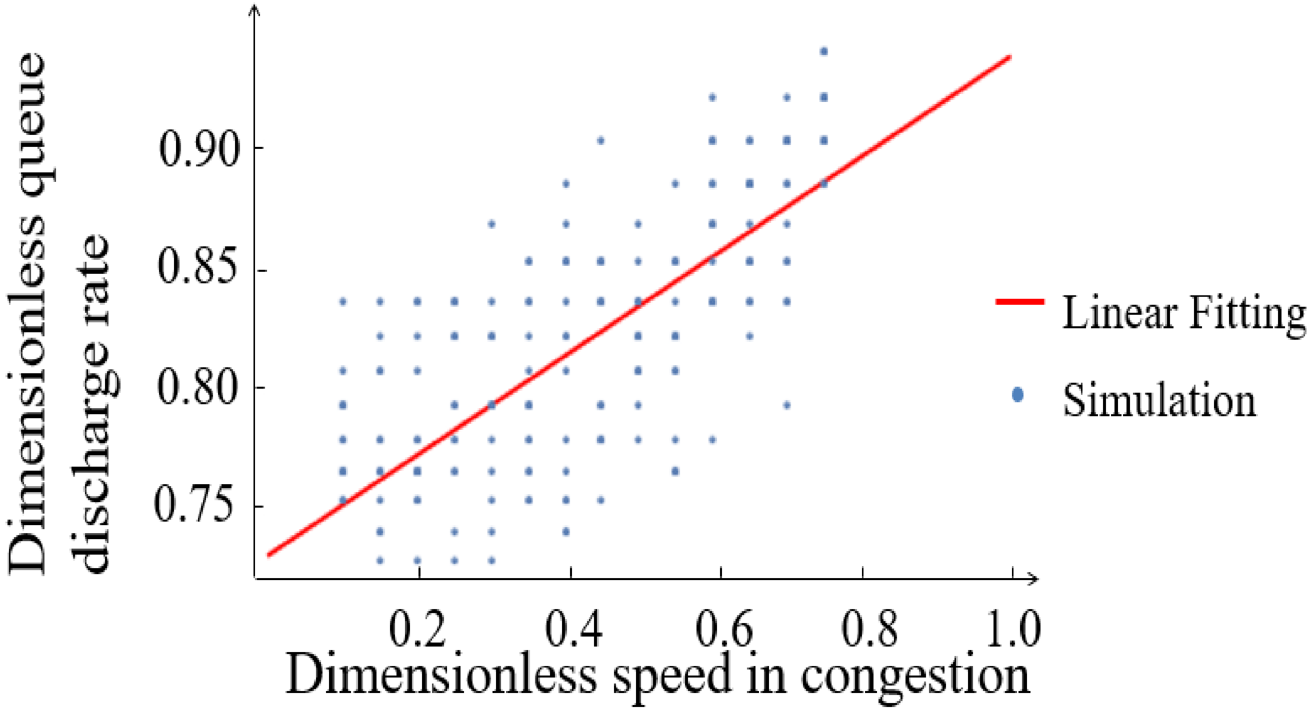
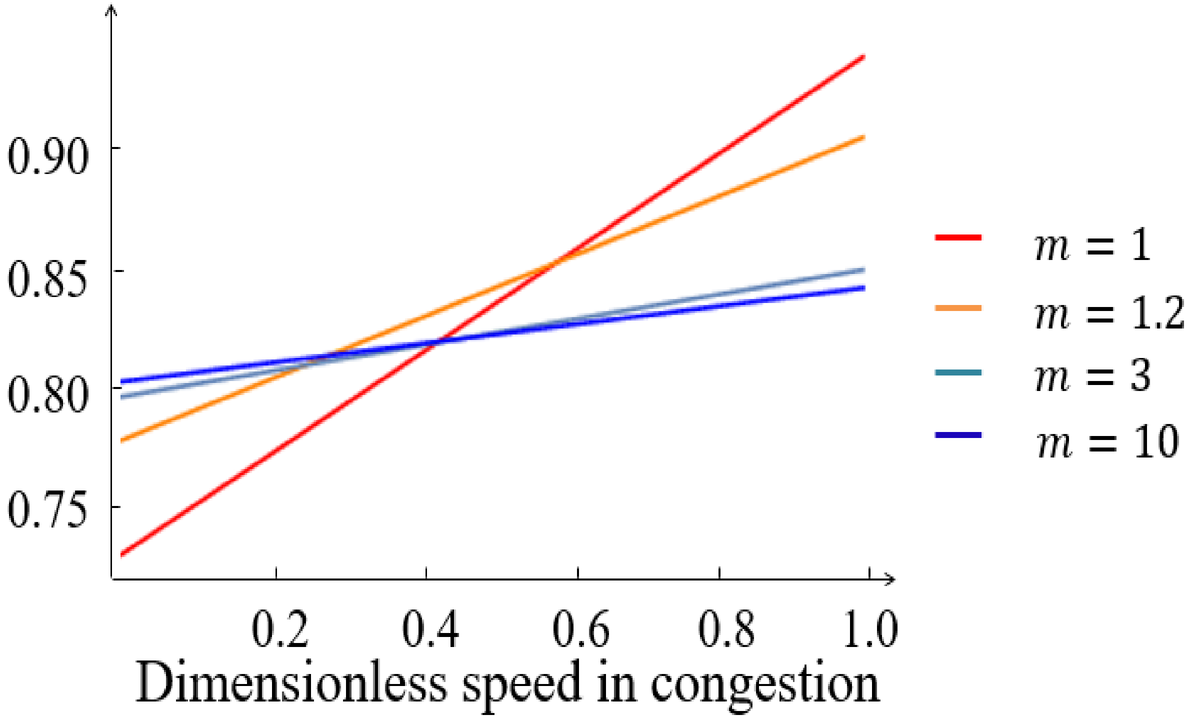


Figure 8 A sample trajectory of the queue discharge experiment. Discharge rate is measured at the back dash line where vehicle speeds reach free-flow speed

4. Speed-capacity relationship at BNs





The model gradually loses its ability to catch the speed-capacity relationship with the increase of the value of m

4. Speed-capacity relationship at BNs

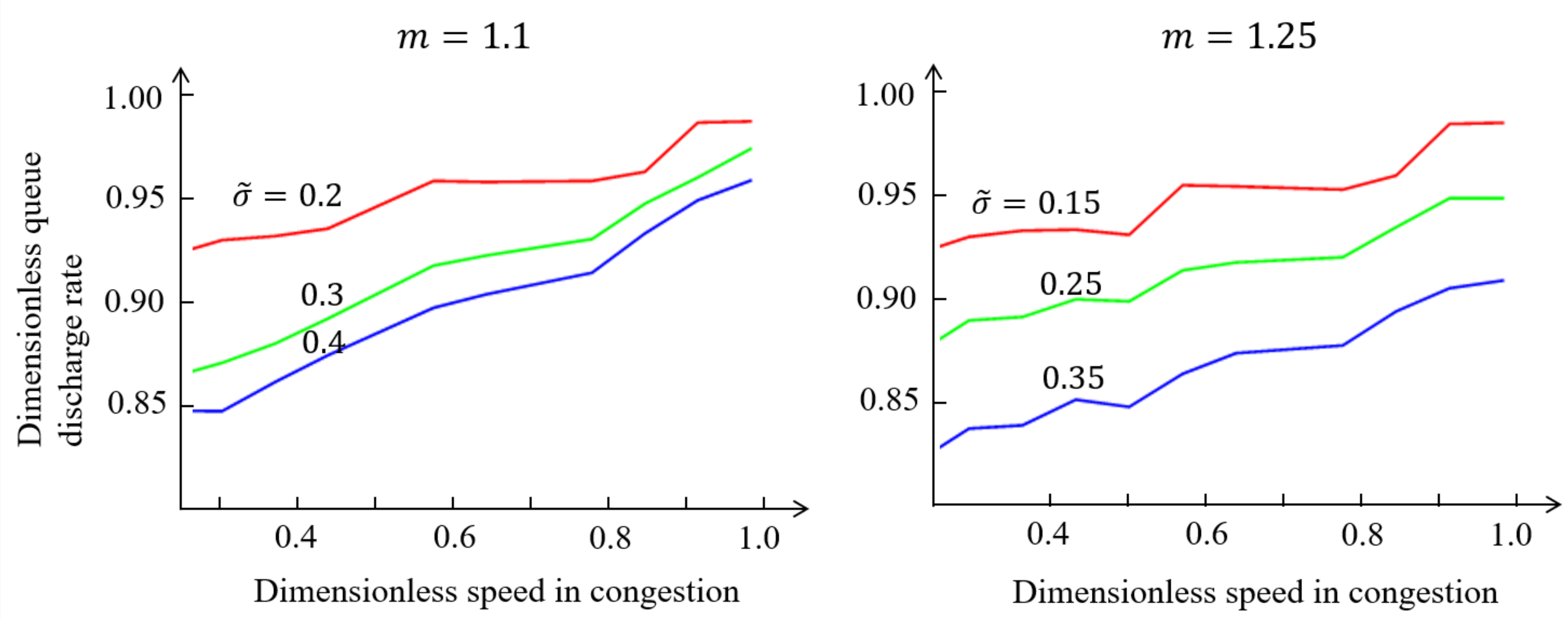
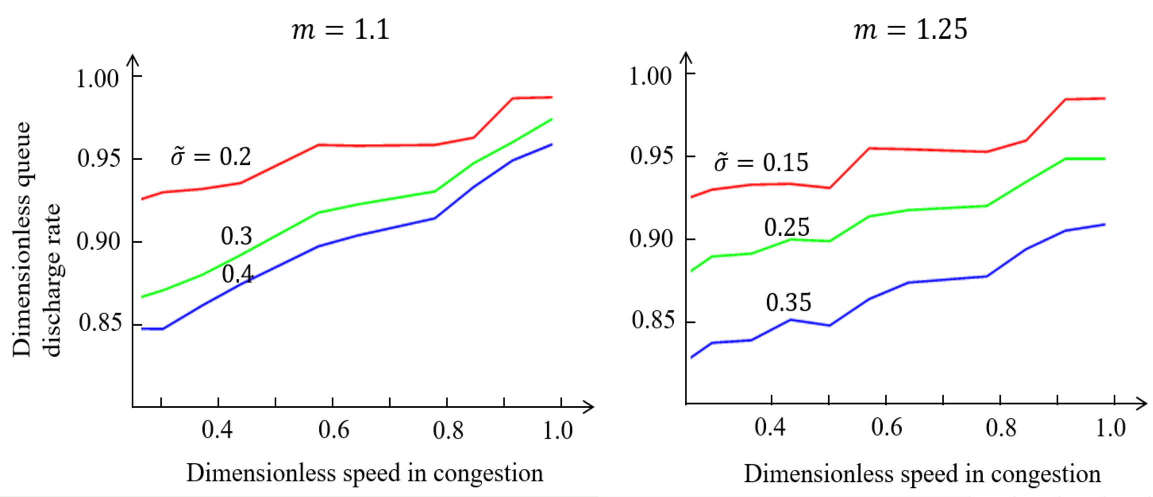
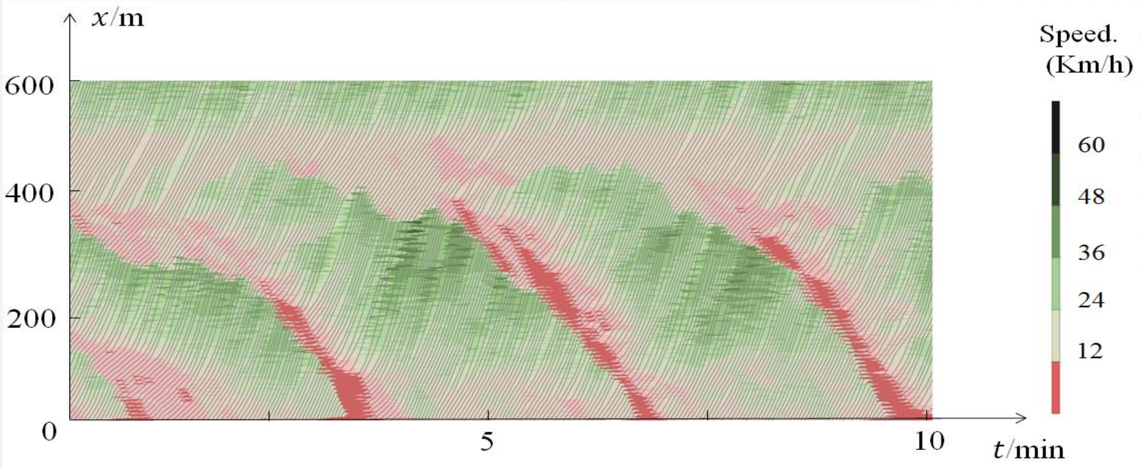


Figure 9 Dimensionless queue discharge rate as a function of speed in congestion for different values of model parameters

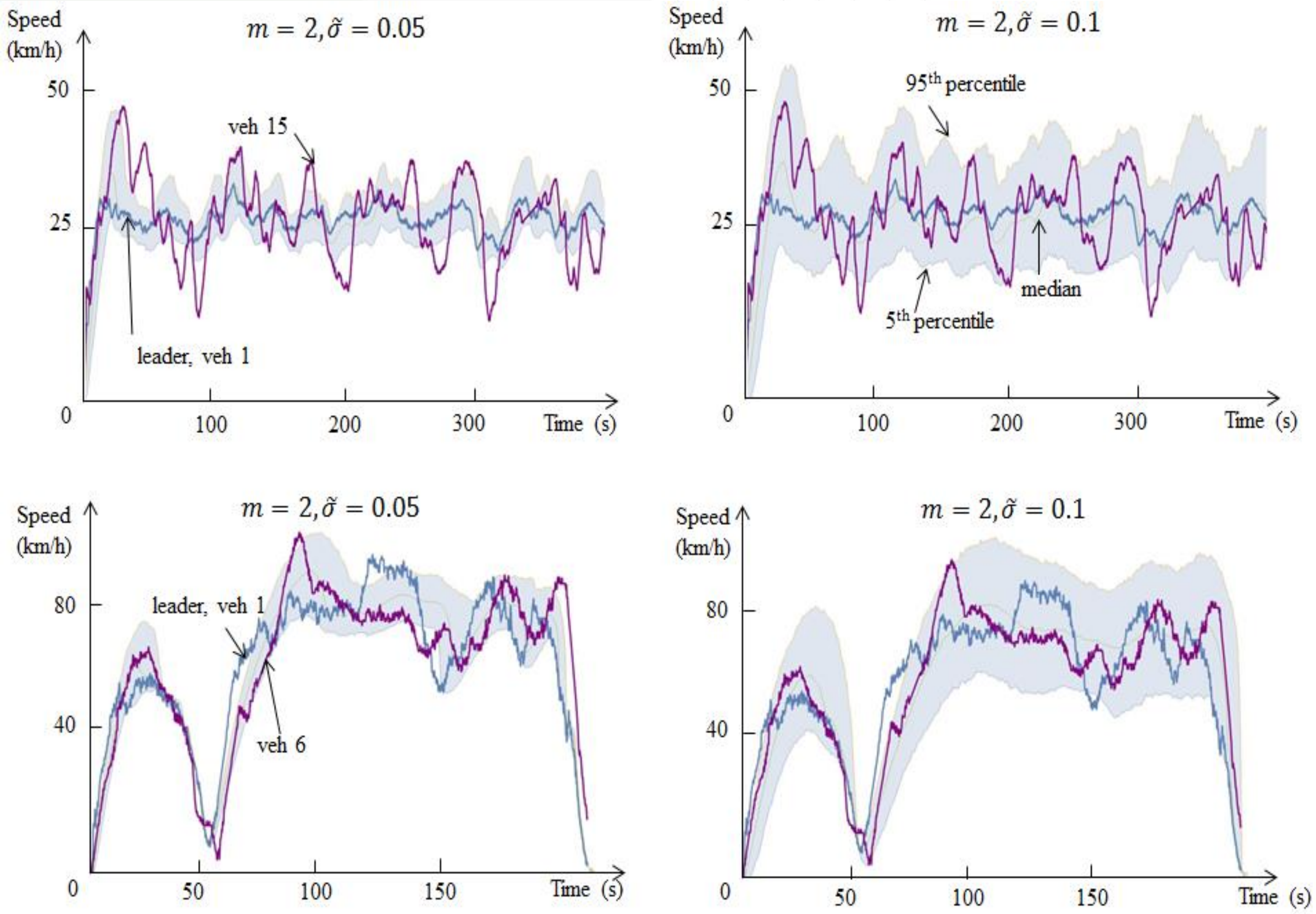
4. Speed-capacity relationship at BNs





A value of $m \approx 1.2$ is able to reproduce both the speed-capacity relationship and realistic traffic oscillations.

5. Vehicle speed distributions



The model predicts the trailing speed distributions well. The width of the probability bands increase with $\tilde{\sigma}$

Conclusion

- We add a unitless parameter to generalize two existing stochastic driver acceleration models. Model parameters can be easily estimated by MLE.
- A suitable value of m, e.g. $m \approx 1.2$, can make the model reproduce speed-capacity relationship and realistic traffic oscillations.
- The product of m and $\tilde{\sigma}$ has a big impact on the model, it determines:

  (i) oscillation period and amplitude

  (ii) stable vehicle index of the concave growth of platoon oscillation

  (iii) average speed at the bottleneck

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